Topology of the Universe from Planck CMB

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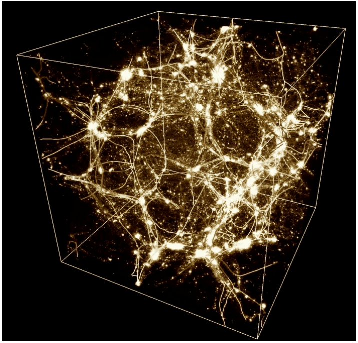
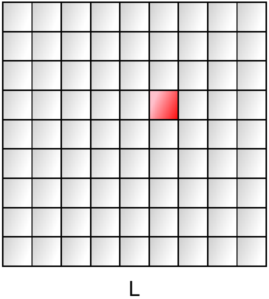
Familiar simulation of the multiconnected toroidal Universe

N-body with periodic boundary conditions



- "Artifacts of the box" at scales $\sim L$
  - Long-wave cutoff in power spectrum
  - Discrete and anisotropic set of wave modes
- No effects of the box at scales $\ll L$

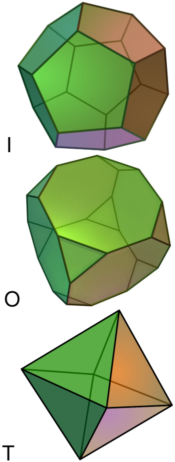
Can be viewed as tiling of infinite space with the copies of the box. This is flat 3-torus space. The box is a "fundamental domain"



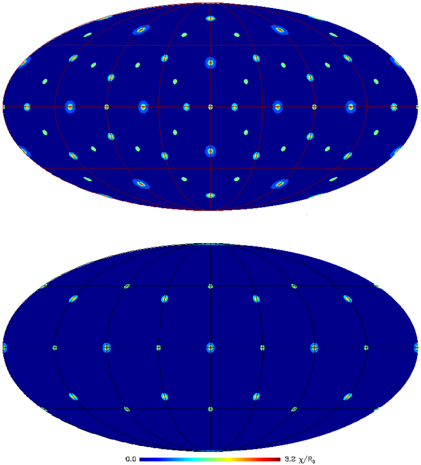
Artifacts of the periodic boundary conditions are observable features if our Universe is indeed 3-torus

CMB provide unique view to large scale organization of the Universe. Can our Universe be multiconnected ?

Tiled with copies of a fundamental domain?



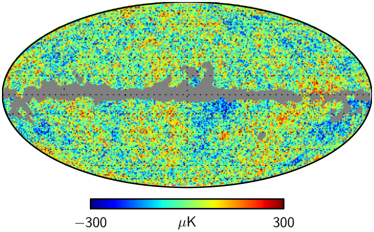
Can we see in different directions repeated images of same objects ?



How far do we see ?

Measurements of Cosmic Microwave Background gives the answer –
up to $\approx 14,000$ Megaparsec (χ_{rec}).
Limited by transparency of our evolving Universe in the past

What we observe from these distances are temperature and polarization maps of CMB photons



Planck collaboration, 2013.
Map of CMB temperature fluctuations.

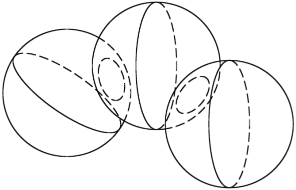
Crucial comparison is of the radius measure of the fundamental domain R_i to the distance travelled by photons χ_{rec}

- $R_i \ll \chi_{rec}$ - photons circumnavigated the Universe multiple times, same regions of space are seem from several distinct directions.
- $R_i \gg \chi_{rec}$ - observed volume is contained entirely within a single fundamental domain, effects of multiconnectivity disappear.

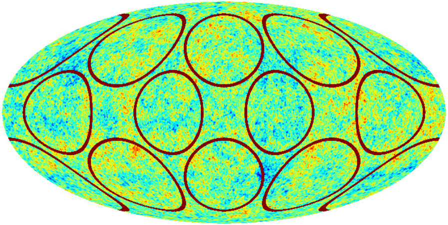
Search for topological signature in CMB maps

Direct search for images

"Circles on the sky"



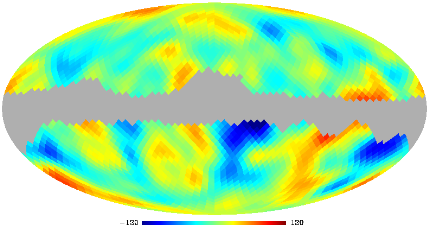
Example for Cubic Torus with $R_i/\chi_{rec} \approx 0.32$



Indirect search

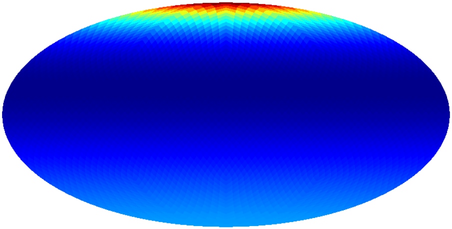
Likelihood analysis of the correlations between the pixels

Planck Smica FWHM=660 arcmin, fsky=0.79



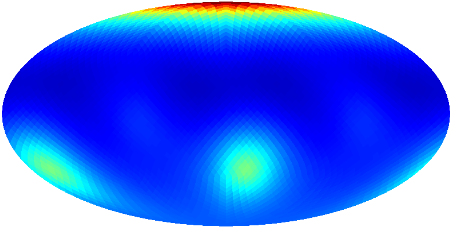
Pixel-pixel correlations and images: Fiducial infinite space

C(North Pole, θ)

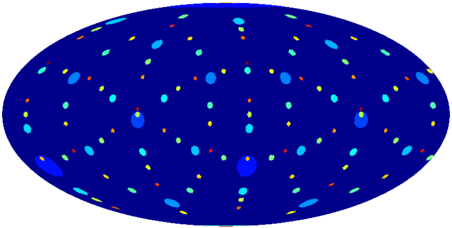


Pixel-pixel correlations and images: χ_{rec} just fits into the compact space

C(North Pole, θ)

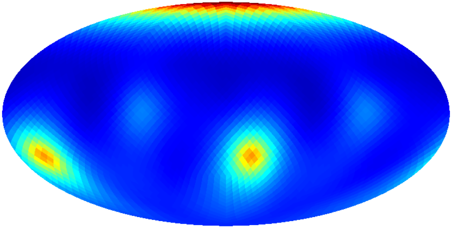


Images of North Pole



Pixel-pixel correlations and images: χ_{rec} just larger than the compact space

C(North Pole, θ)



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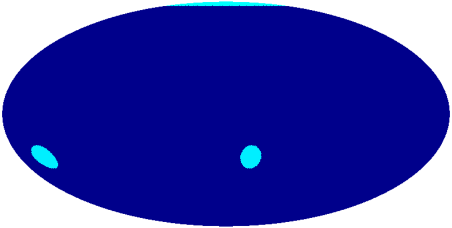

Pixel-pixel correlations and images: χ_{rec} much larger than the compact space

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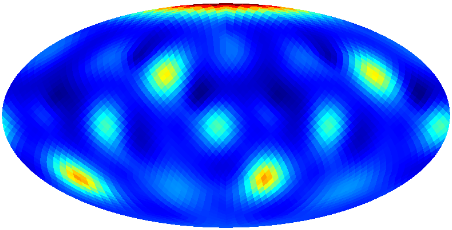
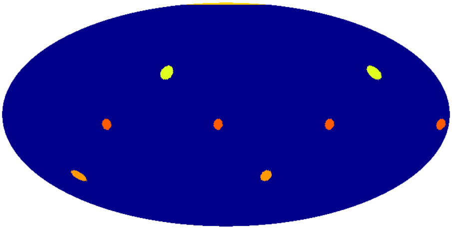


Images of North Pole



Formalism: Likelihood analysis

Log-likelihood

$$\ln(\mathscr{L}) = -\frac{1}{2}\left[n_p \ln(2\pi) + \ln(\det(\mathbf{C}_\mathrm{T} + \mathbf{N})) + \mathbf{x}^\dagger(\mathbf{C}_\mathrm{T} + \mathbf{N})^{-1}\mathbf{x}\right]$$

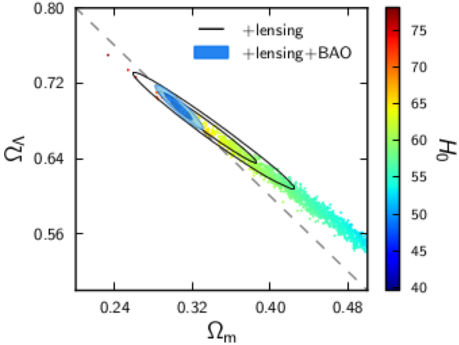
Technical ingredients:

- C_T – theoretical pixel correlation matrix, N – noise correlation function, x – experimental smoothed and masked data.
- Parameters: amplitude of the signal and relative orientation of the data map and theoretical model.
- judicial choice of fiducial modes to project data and model onto is a very important part of the procedure.

Flat and curved FRW models consistent with data still permit Universe to be multiconnected

Flat and closed Universe are allowed by CMB

- $\Omega_k = -0.01^{+0.018}_{-0.019}$ (Planck+lensing+WP+highL)
  $\Omega_k = -0.01$ is a pretty large value - the volume of the sphere is only 100 times the observed volume to LSS.
  Or: the curvature radius $R_0 \approx 3.2\chi_{rec}$
- Small scale fluctuations of the CMB are virtually unchanged if Ω_k tracks degeneracy line.

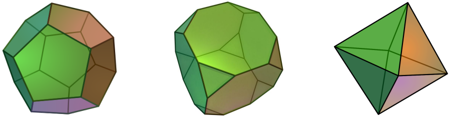


Models analyzed: constant curvature multiconnected spaces

Flat spaces. Size of fundamental domain is continous parameter

Equal and non-equal sides flat $T^3(L_x, L_y, L_z)$ tori, $R_i = L/2 = (0.32 - 1.1) \times \chi_{rec}$.

Spaces of positive curvature. Size of the fundamental domain is linked to the curvature radius



	Dodecahedral	Truncated Cube	Octahedral
$\mathscr{V}/R_0^3$	0.16	0.41	0.82
$\mathscr{R}_\mathrm{i}/R_0$	0.31 ($\pi/10$)	0.39 ($\pi/8$)	0.45
$\mathscr{R}_\mathrm{m}/R_0$	0.37	0.56	0.56
$\mathscr{R}_\mathrm{u}/R_0$	0.40	0.58	0.79 ($\pi/4$)

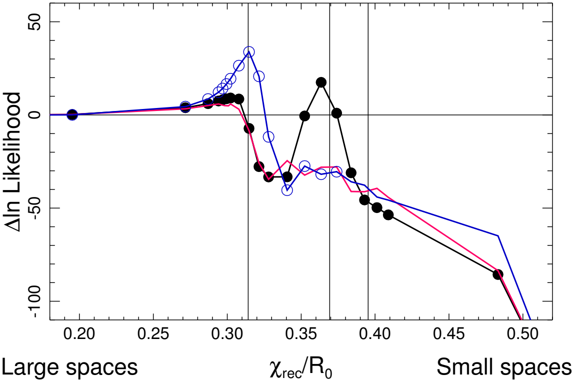
Likelihood detectability of the multiconnected topology

- we compare sequences of multiconnected models that have as its limiting point the fiducial Planck best-fit flat LCDM model
- LnLikelihood is given as the difference with this fixed fiducial model
- For curved spaces we vary the size of the domain by varying the curvature. In addition other parameters as modified to follow the degeneracy line

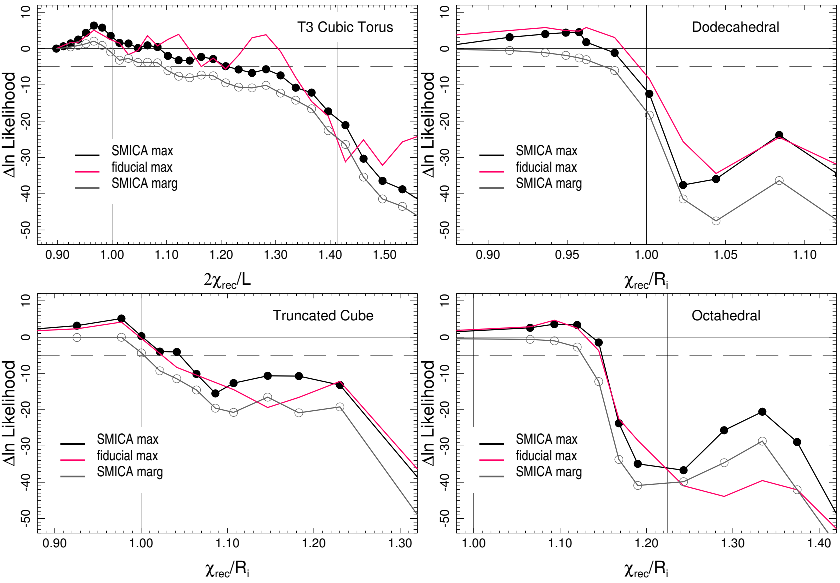
Detecting simulated dodecahedral space

Blue and Black – in two simulated maps indeed drawn from dodecahedral space with two different curvature radii.

Red – in a realization of the fiducial model.



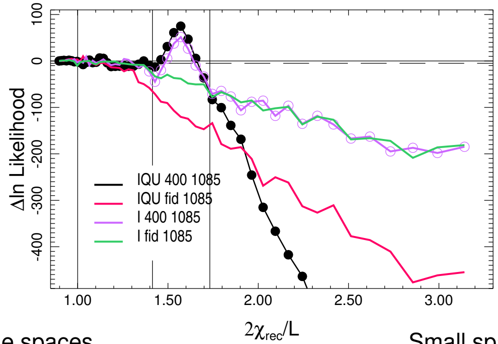
2013 results - no evidence for compact topology



Future advancement – use of polarization

Detecting simulated toroidal space: strengthened detection and/or rejection of small spaces with polarization

- Black and red – using polarization
- Blue and green – temperature only



Large spaces

Small spaces

2013 Lower limits on the size of the fundamental domain for different multiply-connected spaces

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R_i (or $L/2$) $> \chi_{rec}$ means last-scattering sphere fits completely into the fundamental domain of the multiconnected Universe.

Space	Quantity		$\Delta \ln \mathscr{L} < -5$		$\Delta \ln \mathscr{L} < -12.5$	
			max	marg	max	marg
T3 Cubic Torus	$L/(2\chi_{\rm rec})$	$>$	0.83	0.92	0.76	0.83
T2 Chimney	$L/(2\chi_{\rm rec})$	$>$	0.71	0.71	0.63	0.67
T1 Slab	$L/(2\chi_{\rm rec})$	$>$	0.50	0.50	–	–
Dodecahedron	$\mathscr{R}_{\rm i}/\chi_{\rm rec}$	$>$	1.01	1.03	1.00	1.01
Truncated Cube	$\mathscr{R}_{\rm i}/\chi_{\rm rec}$	$>$	0.95	1.00	0.81	0.97
Octahedron	$\mathscr{R}_{\rm i}/\chi_{\rm rec}$	$>$	0.87	0.89	0.87	0.88