Real Space Lensing Reconstruction using CMB Temperature and Polarisation

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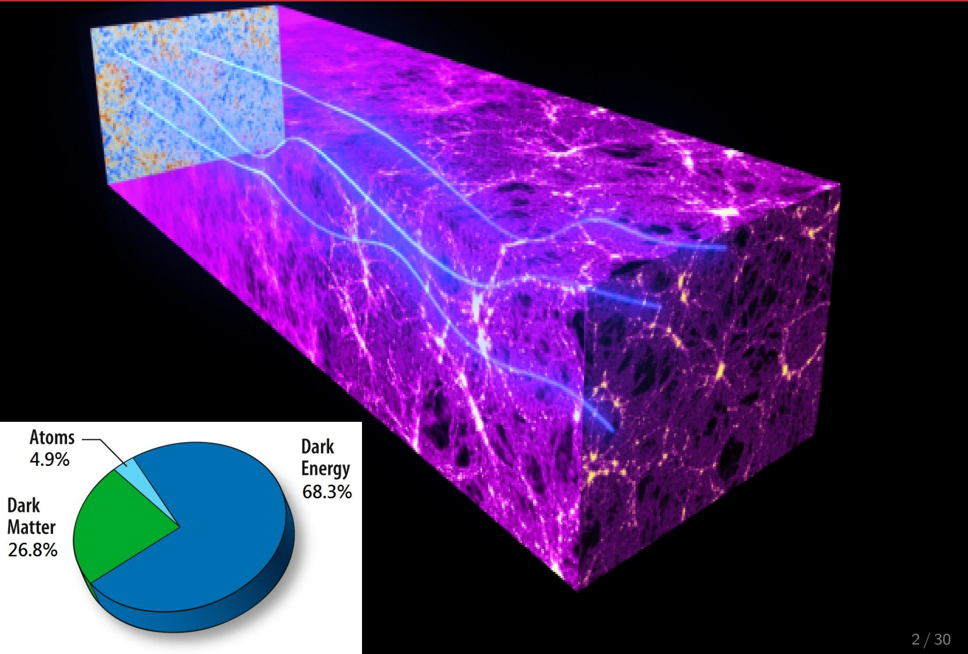
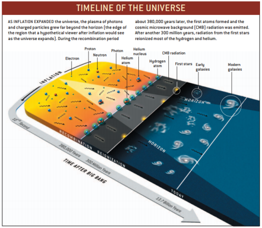
Based on a paper in preparation:
Real Space Lensing Estimation from Polarised CMB maps.
H Prince, J Ridl, K Moodley, M Bucher. 2015

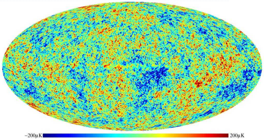
29 January 2014

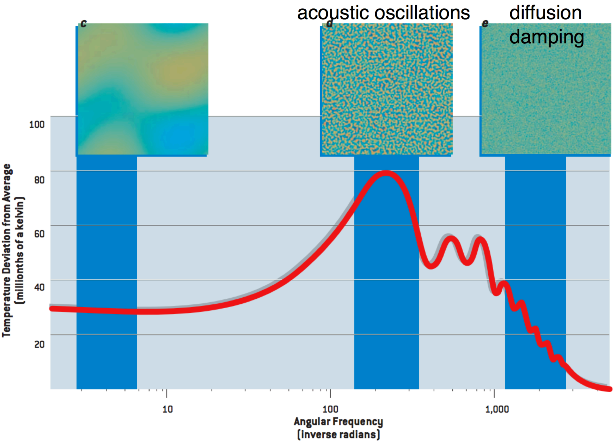
Why do we care about gravitational lensing of the CMB?



CMB Temperature





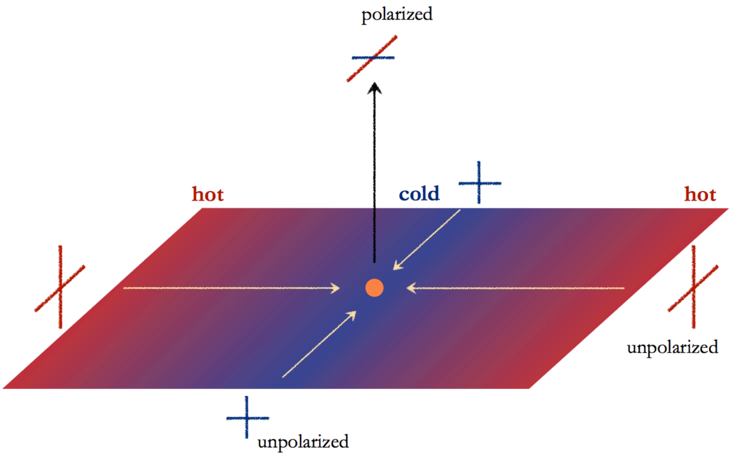


- map is close to isotropic with temperature hot and cold spots at $\mathcal{O}(10^{-5})$
- rich structure of power spectrum allows us to detect effects of lensing

Polarisation from Anisotropy

Thompson Scattering

- quadruple anisotropy present in CMB at last scattering
- Thompson Scattering results in linear polarisation



CMB Polarisation E and B modes

E mode polarisation

- from Thompson Scattering
- radial and tangential around temperature coldspots and hotspots

B mode polarisation

- from gravitational waves and lensed E modes
- makes a 45° angle with the E modes

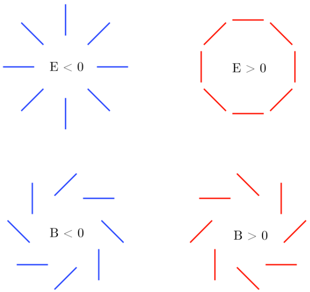


Figure : E and B mode polarisation

Observations of CMB Temperature and Polarisation

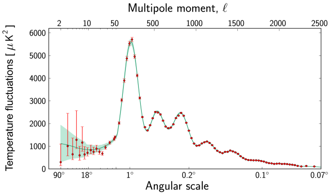
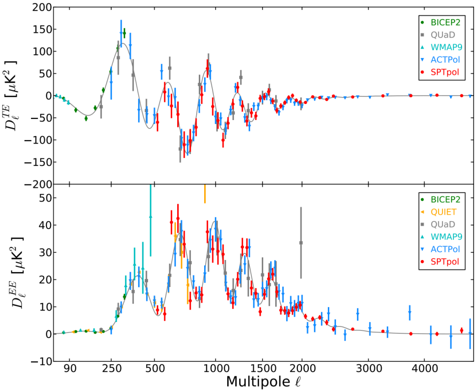


Figure : Planck's CMB Temperature Power Spectrum



E mode and TE power spectra from 2014 SPTPol collaboration paper

Gravitational Lensing

Deflection Angle α

- radiation from direction $\hat{n}$ has been deflected by $\vec{\alpha}$
- $\vec{\alpha}$ is a sum of many small deflections
- lensing remaps photons

$$\underbrace{\tilde{T}(\hat{n})}_{\text{lensed temp}} = \underbrace{T(\hat{n}+\vec{\alpha})}_{\text{unlensed temp}}$$

Lensing Potential ψ

- $\vec{\alpha}(\hat{n}) = \vec{\nabla}\psi(\hat{n})$
- $\psi(\hat{n})$ is the 2D lensing potential (from the 3D matter distribution)

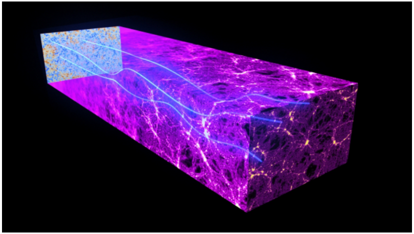
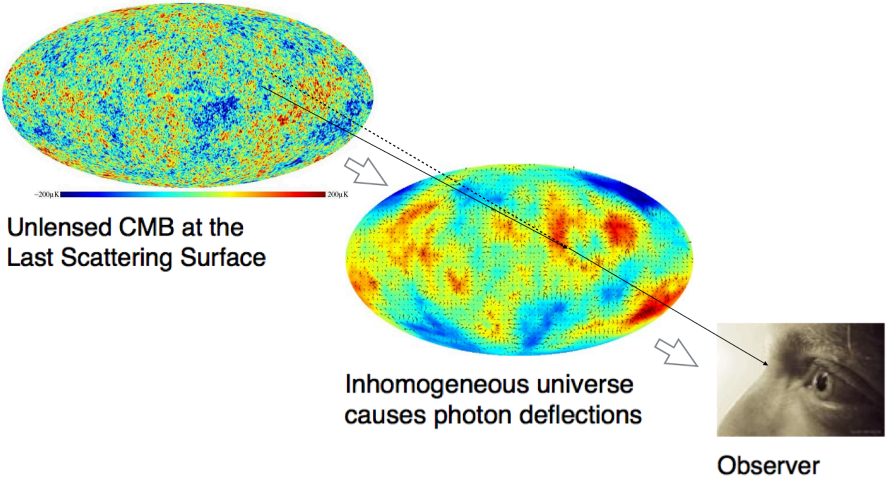


Figure : Deflection of light by mass. The CMB photons undergo multiple deflections adding up to $\vec{\alpha}$

Gravitational Lensing



- average deflection: 2 arcminutes
- coherence scale: 2 degrees

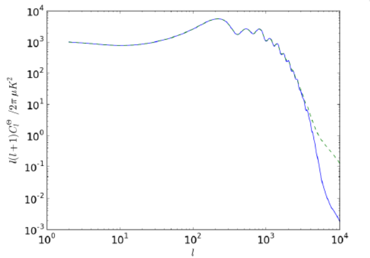
Lensed CMB Temperature Power Spectrum

$$\overbrace{\tilde{T}(\vec{x})}^{\text{lensed}} = \overbrace{T(\vec{x}+\vec{\alpha})}^{\text{unlensed}} = T(\vec{x}+\vec{\nabla}\psi) \approx T(\vec{x}) + (\vec{\nabla}\psi)\cdot(\vec{\nabla}T(\vec{x}))$$

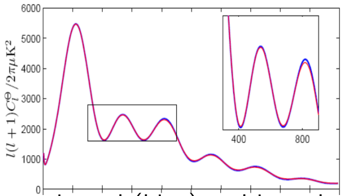
$$\tilde{T}(\vec{l}) \approx T(\vec{l}) - \underbrace{(\vec{l}\psi(\vec{l}))\circ(\vec{l}T(\vec{l}))}_{\text{convolution of gradients}}$$

Effects of Lensing

- peaks of power spectrum spread out
- power transferred to large l



The unlensed (solid) and lensed (dashed) power spectra of the CMB



The unlensed (blue) and lensed (purple) power spectra

Temperature Quadratic Estimator in Harmonic Space

Mode coupling due to lensing is related to ψ

- different Fourier modes of the unlensed CMB are uncorrelated

$$< T(\vec{l}) T^*(\vec{l} - \vec{L}) >_T \approx \delta^D(\vec{L}) c_l^{TT}$$

- lensing induces mode coupling which depends on the lensing potential

$$\overbrace{< \tilde{T}(\vec{l}) \tilde{T}^*(\vec{l} - \vec{L}) >_T}^{\text{lensed}} \approx \overbrace{\delta^D(\vec{L}) c_l^{TT}}^{\text{unlensed}} + \overbrace{\psi(\vec{L})}^{\text{potential}} \times \overbrace{f(\vec{l}, \vec{L})}^{\text{depends on } c_l^{TT}} \qquad (1)$$

Ansatz for ψ

$$\hat{\psi}(\vec{L}) \equiv \underbrace{\frac{1}{N(\vec{L})}}_{\text{normalisation}} \int \frac{d^2\vec{l}}{2\pi} \underbrace{\tilde{T}(\vec{l}) \tilde{T}^*(\vec{l} - \vec{L})}_{\text{lensed temp}} \underbrace{g(\vec{l}, \vec{L})}_{\text{weighting}} \qquad (2)$$

- estimator normalised by $N(\vec{L})$ to be unbiased i.e. $< \hat{\psi}(\vec{L}) >_T = \psi(\vec{L})$
- weighting function $g(\vec{l}, \vec{L})$ minimises the variance

Harmonic Space Estimator

We can rewrite the estimator in terms of two filtered fields F_1 and F_2 as

$$\hat{\psi}(\vec{L}) = -\frac{1}{N(\vec{L})} \int \frac{d^2\vec{x}}{2\pi} e^{-i\vec{L}\cdot\vec{x}} \nabla \cdot [F_1(\vec{x}) \nabla F_2(\vec{x})]$$

- $F_1(\vec{x})$ is related to the small-scale temperature anisotropies
- $\nabla F_2(\vec{x})$ is related to the temperature gradient on large scales

$F_1(\vec{x})$ and $\nabla F_2(\vec{x})$ correlated because on small scales the unlensed CMB is a temperature gradient, and the small-scale anisotropies come from the lensing potential disturbing the gradient

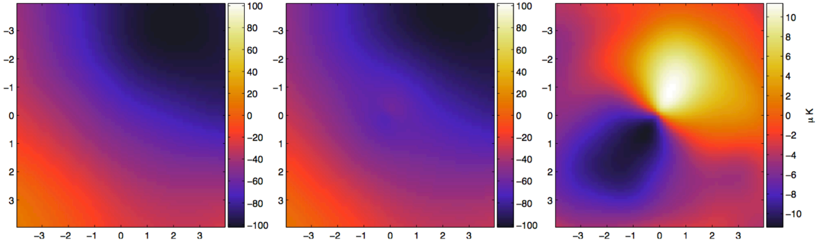


Figure : left: unlensed CMB; middle: lensed CMB; right: difference due to lensing

Why Real Space Estimators?

Harmonic space estimators

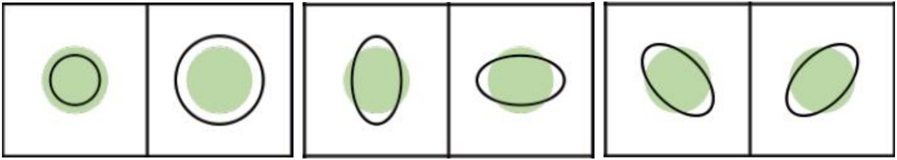
- limitations when it comes to analysing actual experimental data
- implicitly rely on uniform full sky coverage to obtain the Fourier transform

Real-space estimators

- local estimators
- may be sub-optimal
- helpful when analysing experimental data
  - cope with pixels that have been removed from maps
  - cope with non-uniform sky coverage

Real Space Estimators - What to Estimate

- we estimate the convergence κ_0 and two shear components γ_+ and $\gamma_\times$



convergence κ_0 shear γ_+ shear $\gamma_\times$

Assumptions

- $c^{BB}(l) \approx 0$
- lensing fields are large in scale compared to CMB anisotropies

Real Space Estimators

Unlensed Position in terms of Lensed Position

- $\vec{x} = \mathbf{S}\vec{x}\,'$ analogous to $\hat{n} = \hat{n}' + \vec{\alpha}$ from earlier
- $\tilde{T}(\vec{x}\,') = T(\mathbf{S}\vec{x}\,')$ and similarly for polarisation
- $\mathbf{S} = e^{\boldsymbol{\kappa}} \approx \mathbf{I} + \boldsymbol{\kappa}$

Lensed Fields in terms of Unlensed Fields

We use the above relations between lensed and unlensed position in the Fourier Transform equations to obtain:

$$\tilde{T}(\vec{l}) = \det{}^{-\frac{1}{2}}(\mathbf{S})\,T(\mathbf{S}^{-1}\vec{l})$$

$$\tilde{E}(\vec{l}) = \det{}^{-\frac{1}{2}}(\mathbf{S})[E(\vec{l}') + 2(\gamma_\times \cos(2\phi_l) - \gamma_+ \sin(2\phi_l))B(\vec{l}')]$$

$$\tilde{B}(\vec{l}) = \det{}^{-\frac{1}{2}}(\mathbf{S})[B(\vec{l}') - 2(\gamma_\times \cos(2\phi_l) - \gamma_+ \sin(2\phi_l))E(\vec{l}')]$$

where $\vec{l}' = \mathbf{S}^{-1}\vec{l}$ and $\phi_{l'}$ is the angular coordinate of $\vec{l}'$ in polar coordinates.

Real Space Estimators

For $XY{=}TT$, TE and EE, where $\tilde{c}^{XY}(l) =< \tilde{X}^*(\vec{l})\tilde{Y}(\vec{l}) >$, we find:

$$\tilde{c}_l^{XY} = c_l^{XY} + \kappa_0 \times f(c_l^{XY}) + \gamma_+ \cos(2\phi_l) \times g(c_l^{XY}) + \gamma_\times \sin(2\phi_l) \times g(c_l^{XY})$$

Quadratic Estimators for $XY{=}TT$, TE and EE

Ansatz for convergence estimator:

$$\hat{\kappa}_0^{XY} = \underbrace{\frac{1}{N_{\hat{\kappa}_0}^{XY}}}_{\text{normalisation}} \int d^2\vec{l} \left(\underbrace{\tilde{X}^*(\vec{l})\tilde{Y}(\vec{l})}_{\text{lensed fields}} - c_l^{XY} \right) \underbrace{g_{\hat{\kappa}_0}^{XY}(l)}_{\text{weighting}} \tag{3}$$

Ansatz for shear estimator:

$$\begin{Bmatrix} \hat{\gamma}_+^{XY} \\ \hat{\gamma}_\times^{XY} \end{Bmatrix} = \underbrace{\frac{1}{N_{\hat{\gamma}_+,\hat{\gamma}_\times}^{XY}}}_{\text{normalisation}} \int d^2\vec{l}\, \underbrace{\tilde{X}^*(\vec{l})\tilde{Y}(\vec{l})}_{\text{lensed fields}} \begin{Bmatrix} \cos(2\phi_{lL}) \\ \sin(2\phi_{lL}) \end{Bmatrix} \underbrace{g_{\hat{\gamma}_+,\hat{\gamma}_\times}^{XY}(l)}_{\text{weighting}} \tag{4}$$

Real Space Estimators

For $Y = T$ or E:

$$\tilde{c}^{YB}(l) = < \tilde{Y}^*(\vec{l})\tilde{B}(\vec{l}) > = 2c^{YE}(l)\left[\gamma_+ \sin(2\phi_l) - \gamma_\times \cos(2\phi_l)\right]$$

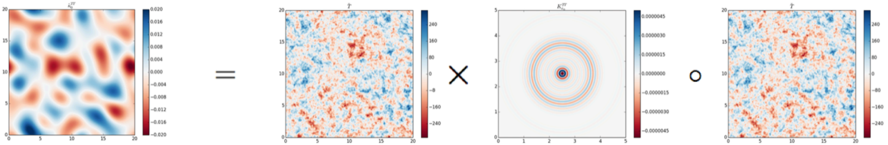
Quadratic Estimators for TB and EB

Ansatz for shear estimator:

$$\begin{Bmatrix} \hat{\gamma}_+^{YB} \\ \hat{\gamma}_\times^{YB} \end{Bmatrix} = \underbrace{\frac{1}{N^{YB}_{\hat{\gamma}_+,\hat{\gamma}_\times}}}_{\text{normalisation}} \int d^2\vec{l}\, \underbrace{\tilde{Y}^*(\vec{l})\tilde{B}(\vec{l})}_{\text{lensed fields}} \begin{Bmatrix} \sin(2\phi_{lL}) \\ \cos(2\phi_{lL}) \end{Bmatrix} \underbrace{g^{YB}_{\hat{\gamma}_+,\hat{\gamma}_\times}(l)}_{\text{weighting}} \qquad (5)$$

Real Space Estimators - Implementation

$$\hat{\kappa}_0^{TT}(\vec{x}) = \tilde{T}(\vec{x}) \times \left(K_{\hat{\kappa}_0}^{TT}(\vec{x}) \circ \tilde{T}(\vec{x}) \right)$$



where $\circ$ denotes convolution.

Equivalent expressions can be found for the other estimators.

Real Space Estimator and the Harmonic Space Estimator

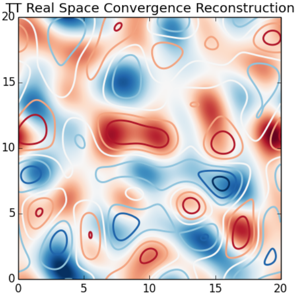
Low-L (large-scale lensing field) limit of harmonic space estimator

- Taking the low-L limit of the harmonic space estimator (left) gives us the inverse-variance weighting of the real space convergence and shear estimators (right)

$$\lim_{L\to 0}\left(\frac{1}{2}L_i L_j \hat{\psi}^{HS}(\vec{L})\right) = \frac{N_{\hat{\kappa}_0}}{N_{\hat{\kappa}_0}+N_{\hat{\gamma}_+}}\hat{\kappa}_0^{RS} + \frac{N_{\hat{\gamma}_+}}{N_{\hat{\kappa}_0}+N_{\hat{\gamma}_+}}\hat{\gamma}_+^{RS} \tag{6}$$

Real Space Lensing Reconstructions - Convergence from Temperature

- contours represent input convergence, colour maps show reconstructed convergence



κ_0 from 1 map

Real Space Lensing Reconstructions - Convergence from Temperature

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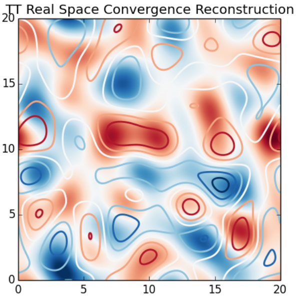
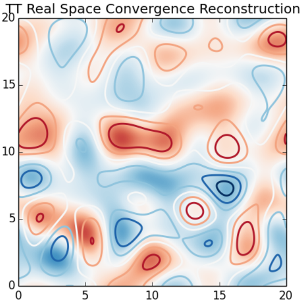


κ_0 from 1 map



κ_0 from 1 map minus noise

Real Space Lensing Reconstructions - Convergence from Temperature

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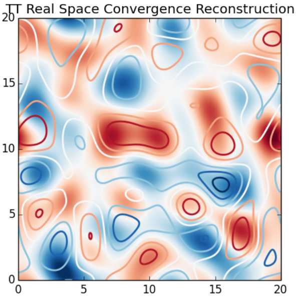
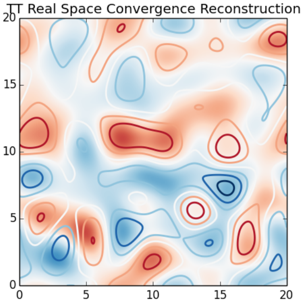
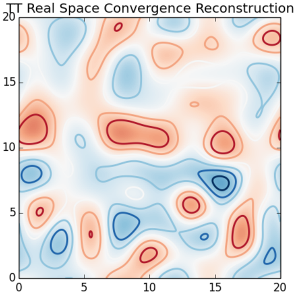


κ_0 from 1 map

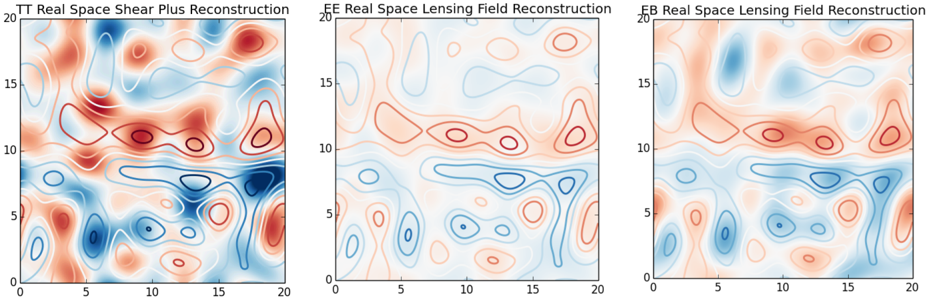


κ_0 from 1 map minus noise



κ_0 from 20 maps

Real Space Lensing Reconstructions - Shear Plus

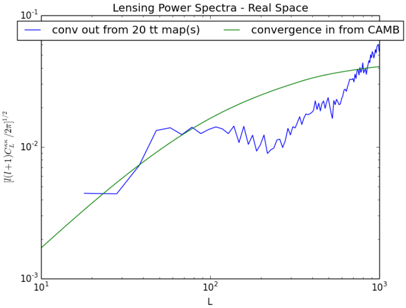


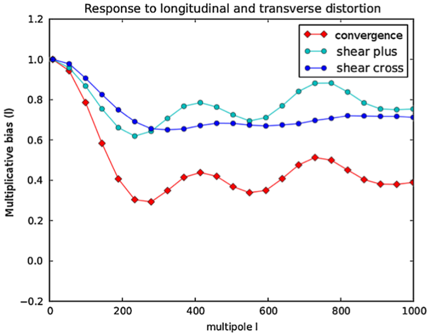
From left to right: TT, EE and EB shear plus reconstructions

EB Estimator

- best reconstruction
- no noise variance from unlensed B field

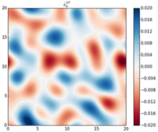
Real Space Lensing Reconstructions - Convergence from Temperature





Improving High-L Reconstruction

$$\hat{\kappa}_0^{TT}(\vec{x}) = K_1 \times \Big(\big(K_2 \circ \tilde{T}(\vec{x}) \big) \times \big(K_3 \circ \tilde{T}(\vec{x}) \big) \Big)$$



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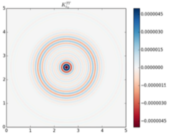

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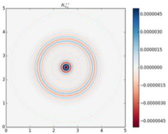

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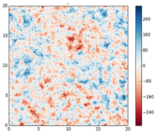

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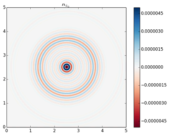

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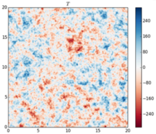


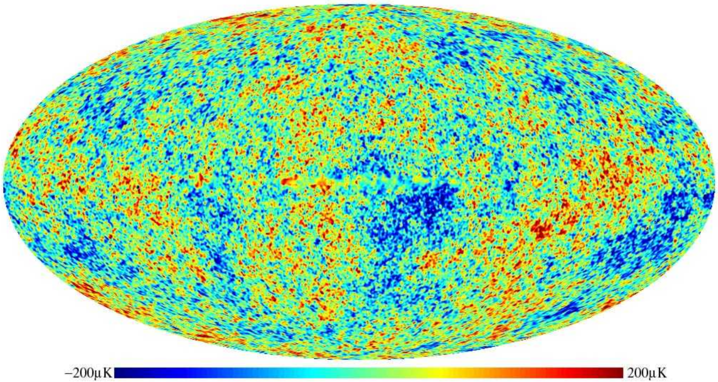
Conclusion

Applications of Lensing

- mapping the distribution of matter
- multiple estimators give reconstructions that can be compared

Next Steps

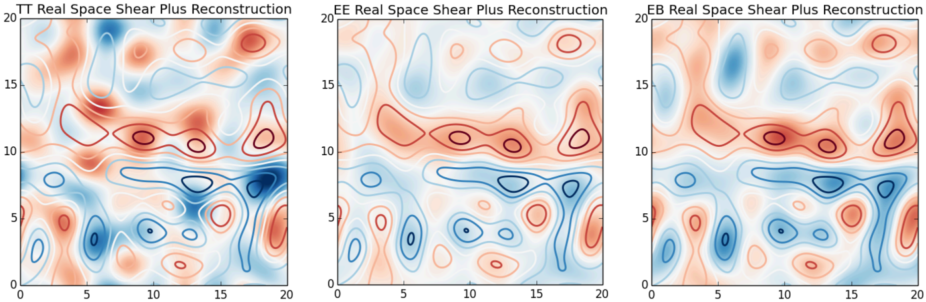
- apply to ACTPol maps
- extend beyond slowly-varying lensing field approximation



The End

Real Space Lensing Reconstructions - Shear Plus

Noise for reference experiment similar to ACTPol

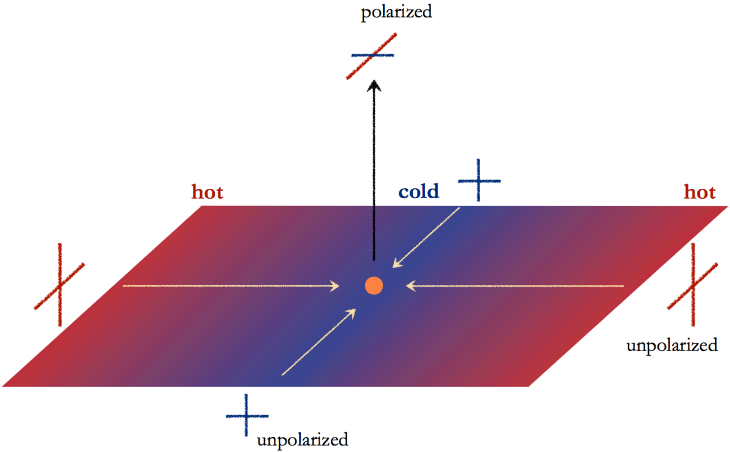


From left to right: TT, EE and EB shear plus reconstructions

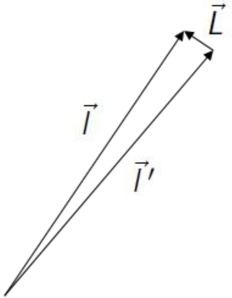
Polarisation from Anisotropy

Thompson Scattering

- quadruple anisotropy present in CMB at last scattering
- Thompson Scattering results in linear polarisation



Real Space Estimators -Squeezed Triangle Approximation



- $|\vec{l}| \approx |\vec{l}'|$, both large (small scale CMB anisotropies) where $\vec{l}$ and $\vec{l}'$ are the unlensed and lensed wavevectors
- $|\vec{L}| = |\vec{l} - \vec{l}'| << |\vec{l}|$ (large scale lensing potential)

- large scale lensing fields (small L)
- slowly varying κ_0, γ_+ and $\gamma_\times$

So we will work with areas of the sky over which κ_0, γ_+ and $\gamma_\times$ are approximately constant.

Real Space Estimators

Unlensed Position in terms of Lensed Position

- $\vec{x} = \mathbf{S}\vec{x}\,'$ analogous to $\hat{n} = \hat{n}' + \vec{\alpha}$ from earlier
- $\tilde{T}(\vec{x}\,') = T(\mathbf{S}\vec{x}\,')$ and similarly for polarisation
- $\mathbf{S} = e^{\boldsymbol{\kappa}} \approx \mathbf{I} + \boldsymbol{\kappa}$

Lensed Fields in terms of Unlensed Fields

We use the above relations between lensed and unlensed position in the Fourier Transform equations to obtain:

$$\tilde{T}(\vec{l}) = \det{}^{-\frac{1}{2}}(\mathbf{S})\, T(\mathbf{S}^{-1}\vec{l})$$

$$\tilde{E}(\vec{l}) = \det{}^{-\frac{1}{2}}(\mathbf{S})[E(\vec{l}') + 2(\gamma_\times \cos(2\phi_l) - \gamma_+ \sin(2\phi_l))B(\vec{l}')]$$

$$\tilde{B}(\vec{l}) = \det{}^{-\frac{1}{2}}(\mathbf{S})[B(\vec{l}') - 2(\gamma_\times \cos(2\phi_l) - \gamma_+ \sin(2\phi_l))E(\vec{l}')]$$

where $\vec{l}' = \mathbf{S}^{-1}\vec{l}$ and $\phi_{l'}$ is the angular coordinate of $\vec{l}'$ in polar coordinates.

Real Space Estimators - Lensed Spectra in terms of Unlensed Spectra

For the unlensed spectra,

- we assume $c^{BB}(l) \approx 0$ – we clearly need to extend our approach to take primordial B modes into account
- $c^{EB}(l) = 0 = c^{TB}(l)$ by parity considerations

We use the lensed fields to find the lensed power spectra.
For $XY{=}TT$, TE and EE, where $\tilde{c}^{XY}(l) =< \tilde{X}^*(\vec{l})\tilde{Y}(\vec{l}) >$, we find:

$$\boxed{\begin{aligned}\tilde{c}^{XY}(l) =& c^{XY}(l)\left[1 - \kappa_0\left(\frac{d\ln[c^{XY}(l)]}{d\ln[l]} + 2\right)\right] \\ &- c^{XY}(l)(\gamma_+\cos(2\phi_l) + \gamma_\times\sin(2\phi_l))\frac{d\ln[c^{XY}(l)]}{d\ln[l]}\end{aligned}}$$

For $Y = T$ or E:

$$\boxed{\tilde{c}^{YB}(l) =< \tilde{Y}^*(\vec{l})\tilde{B}(\vec{l}) >= -2c^{YE}(l)\left[\gamma_\times\cos(2\phi_l) - \gamma_+\sin(2\phi_l)\right]}$$

$\boxed{\tilde{c}^{BB}(l) = 0}$ in our current approximation.

Real Space Convergence Estimators

- Ansatz for $XY = TT$, TE and EE:

$$\hat{\kappa}_0^{XY} = \frac{1}{N_{\hat{\kappa}_0}^{XY}} \mathcal{A} \int \frac{d^2\vec{l}}{(2\pi)^2} \left(\tilde{X}^*(\vec{l}) \tilde{Y}(\vec{l}) - c_l^{XY} \right) g^{XY}(l)$$

- $N_{\hat{\kappa}_0}^{XY}$ is the normalisation constant, found by assuming that the estimator is unbiased, i.e. $< \hat{\kappa}_0^{XY} >_T = \kappa_0$
- $g^{XY}(\vec{l})$ is a weighting function, found by minimising the variance
- We subtract the unlensed power spectrum c_l^{XY} from the observed one to isolate κ_0

Real Space Shear Estimators for $XY = TT$, TE and EE

We multiply by $\cos(2\phi_l)$ and $\sin(2\phi_l)$ in the ansatz to isolate the γ_+ and $\gamma_\times$ parts respectively:

$$\begin{pmatrix} \hat{\gamma}_+^{XY} \\ \hat{\gamma}_\times^{XY} \end{pmatrix} = \frac{1}{N^{XY}_{\hat{\gamma}_+,\hat{\gamma}_\times}} \int \frac{d^2\vec{l}}{(2\pi)^2} F^{XY} \left(\frac{d\ln[c^{XY}(l)]}{d\ln[l]} \right) \begin{pmatrix} \cos(2\phi_l) \\ \sin(2\phi_l) \end{pmatrix} \tilde{X}^*(\vec{l})\tilde{Y}(\vec{l})$$

where

$$N^{XY}_{\hat{\gamma}_+\hat{\gamma}_\times} = \frac{1}{2} \int \frac{d^2\vec{l}}{(2\pi)^2} F^{XY} c^{XY}(l) \left(\frac{d\ln[c^{XY}(l)]}{d\ln[l]} \right)^2$$

and $F^{XY} = \frac{c^{XY}(l)}{(c^{XY}(l)+n^{XY}(l))^2}$ if $XY = TT$ or EE and
$F^{TE} = \frac{c^{TE}(l)}{(\tilde{c}^{TE}(l))^2+(c^{TT}(l)+n^{TT}(l))(c^{EE}(l)+n^{EE}(l))}$

Real Space Shear Estimators for $XY = TB$ and EB

Can use $\tilde{c}^{TB}(l)$ and $\tilde{c}^{EB}(l)$ to find estimators for γ_+ and $\gamma_\times$. In the following Y denotes either T or E

$$\begin{pmatrix} \hat{\gamma}_+^{YB} \\ \hat{\gamma}_\times^{YB} \end{pmatrix} = \frac{1}{N^{YB}_{\hat{\gamma}_+,\hat{\gamma}_\times}} \int \frac{d^2\vec{l}}{(2\pi)^2} \left(\frac{c^{YE}(l)}{(c^{YY}(l) + n^{YY}(l))(n^{BB}(l))} \right) \times \\ \times \begin{pmatrix} \sin(2\phi_l) \\ \cos(2\phi_l) \end{pmatrix} \tilde{Y}^*(\vec{l})\tilde{B}(\vec{l})$$

Where

$$N^{YB}_{\hat{\gamma}_+\hat{\gamma}_\times} = \int \frac{d^2\vec{l}}{(2\pi)^2} \frac{(c^{YE}(l))^2}{(c^{YY}(l) + n^{YY}(l))(n^{BB}(l))}$$

Real Space Estimators - Implementation

The convergence estimator $\hat{\kappa}_0$ can be used to find $\hat{\kappa}_0^{TT}(\vec{x})$ in terms of

- the real space temperature field $T(\vec{x})$
- a filter $K_{\hat{\kappa}_0}^{TT}$ (which is related to the Fourier transform of the weight function $g^{TT}(l)$)

as

$$\hat{\kappa}_0^{TT}(\vec{x}) = T(\vec{x})(K_{\hat{\kappa}_0}^{TT} \circ T)(\vec{x}) - < T(\vec{x})(K_{\hat{\kappa}_0}^{TT} \circ T)(\vec{x}) >_{\text{unlensed}} \quad (7)$$

where $\circ$ denotes convolution.
Similar expressions can be found for $\hat{\gamma}_+^{TT}(\vec{x})$ and $\hat{\gamma}_\times^{TT}(\vec{x})$, and for the polarisation and cross estimators.

How good is our squeezed triangle approximation?

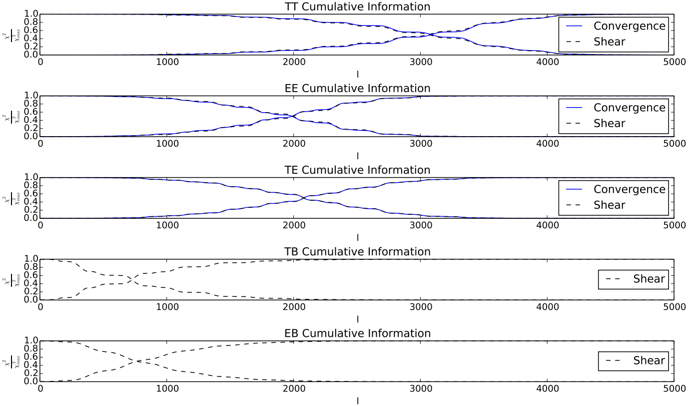
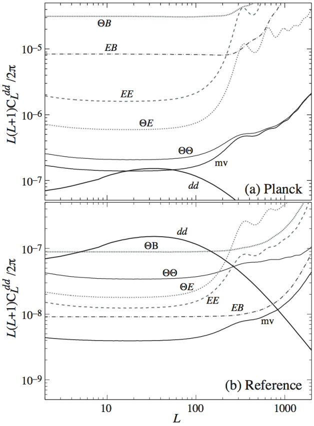


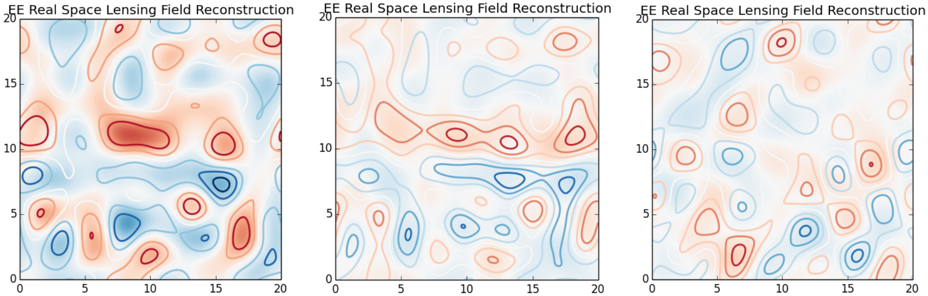
Figure : Normalized cumulative χ^2 (or equivalently N) as a function of l integrated both from the left and from the right using the sensitivity and resolution parameters for the ACT experiment

Which estimators are the best?

Best estimator depends on experiment

The plot shows the deflection signal (dd) and noise power spectra of the quadratic estimators and their minimum variance (mv) combination. As the sensitivity of the experiment improves the best quadratic estimator switches from TT to EB.





Convergence, Shear Plus and Shear Cross Reconstructions from E mode polarisation maps.

Real Space Lensing Reconstructions - Convergence from Temperature